AP Physics C: E&M Summary

Michel Liao*

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*All of these notes come from College Board's AP Daily Videos unless otherwise stated.

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1 Notes

These notes are not comprehensive. Namely, derivations for RC, LC, and LR circuits are not included in this document. Those concepts are important to know, however.

All boxed equations are on your equation sheet.

Please email me at michel.liao@systemgreen.org if you have any questions or notice any typos.

2 Things to Practice

- Gauss's Law applications
- Deriving charging/discharging a capacitor in an RC circuit
  - Charging a Capacitor in an RC Circuit:
  $$q = C\varepsilon - C\varepsilon e^{-\frac{t}{RC}} = Q_0 - Q_0 e^{\frac{-t}{RC}}.$$
  - Discharging a Capacitor in an RC Circuit:
  $$q = Q_0 e^{\frac{-t}{RC}}.$$
- Deriving charging/discharging a capacitor in an LC circuit
- Charging RL Circuit:
$$I_f - I_f e^{-\frac{R}{L}t}.$$
- Discharging RL Circuit:
$$I_0 e^{-\frac{R}{L}t}.$$
- Period for LC circuit:
$$2\pi\sqrt{LC}.$$

3 Electrostatics

3.1 Charge and Coulomb's Law

3.1.1 Introduction

- The **Fundamental Law of Charges** says that opposite charges attract and similar charges repel.

- $$\boxed{\left|\vec{F_E}\right| = \frac{1}{4\pi\epsilon_0}\left|\frac{q_1 q_2}{r^2}\right|}.$$ [1]

- Use vector addition to solve 2-D problems.

3.1.2 Methods of Charging

- Charging by friction:
  - You can rub neutral silk with a neutral glass rod, transferring electrons from the rod to the silk.
- Charging by conduction/contact/touching:
  - Electrons transfer when a charged object touches a neutral object.
- Charging by induction (only conductors can be charged by this process):
  1. Bring a charged object close to the conductor.
  2. Touch the conductor with a ground or a second conductor.
  3. The conductor now has a net charge.

3.1.3 Insulators and Conductors

- **Insulators** impede the flow of electrons. **Conductors** allow electrons to flow freely.
- Neutral insulators can **polarize**, causing the electrons to reorient themselves.
- Conductors undergo **induced charge separation**, where the electrons *move* instead of just reorienting.

3.2 Electric Field and Electric Potential

3.2.1 Electric Field

- An electric field is created by charged objects. The field causes electric forces on other charges.
- Diagram the electric field as if it were putting a force on a *positive* test charge.
  - With a negative test charge, the electric field is drawn "backwards." In other words, the arrows go the opposite ways.

[1] This equation only tells you the *magnitude* of the electric force. Use logic to find the direction.

- $$\boxed{\vec{E} = \frac{\vec{F}_E}{q}}.$$
  - Note that q is our test charge.
  - If we substitute $F_E = k\frac{Qq}{r^2}$, where Q is the source charge and q is the test charge, we get the following: $E = \frac{kQ}{r^2}$.
- Solve static-force problems in uniform electric fields by:
  1. Free body diagram
  2. $\sum F = 0$ (in two directions if needed)
  3. Algebra
- Charged particle motion through uniform fields is parabolic given an inital velocity perpendicular to the field. Combine forces with kinematics to solve the problem.
  - Note that we can often neglect F_g because $F_g \ll F_E$.

3.3 Electric Potential Energy

3.3.1 Introduction

- Electric potential energy is not the same as potential difference.
- **Electric potential energy** is the energy stored in the arrangement of charges.
  - The negative of the work done by a conservative force (electric field) to arrange the charges
  - It's not about an object, it's a property of a system of objects
  - $$\boxed{U_E = qV = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}}.$$
  - We can also write this as $W = q\Delta V$.
- We always talk about U_E in differences. If there is no Δ, we assume $U_E = 0J$ at infinity.
- When finding the potential energy of a system, pretend there is nothing at first and then bring in each charge from infinity.

3.3.2 Conservation of Energy

- Electric force is a conservative force, so mechanical energy is conserved.
- Conservation of energy can be used to determine the changes in speed of the particles in a system.

3.4 Electric Potential

- Electric potential is measured in Volts.
  - Electric potential is also called potential, potential difference, and difference.
  - It is *not* electric potential energy
- **Potential** is the amount of potential energy per unit charge that a charged object would feel at a location.
  - Scalar
  - Always a comparison to the potential somewhere else (usually 0 V at infinity)
  - $$\boxed{\Delta V = -\int \vec{E} \cdot d\vec{r}}.$$
  - $$\boxed{V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}}.$$
  - Outside of a point charge, $V = k\frac{q}{r}$.
- **Equipotential lines** are lines that show places that have the same potential.
  - The electric field is always perpendicular to equipotential lines and points toward lower potentials.
- Positive charges go to lower potentials. Negative charges go to higher potentials.
- When a charge moves *perpendicular* to a field, no work is done. When a charge moves *parallel* to a field, work is done by the electric force.
- Positive charges lose potential energy when they move to a lower potential.
- Since electric force is conservative, any closed loop path requires 0 work and 0 change in potential.
- $$\boxed{E_x = -\frac{dV}{dx}}.$$
  - This tells us that the electric field is greatest when equipotential lines are closer together.
- $W_{\text{ext force}} = \Delta U$ while $W_E = -\Delta U$

3.5 Gauss's Law

3.5.1 Introduction

- **Electric flux** is the amount of field lines flowing through an area.
-
$$\Phi = \vec{E} \cdot \vec{A}.$$ [2,3]
  - Otherwise written as $\Phi = |E| \cos\theta |A|$.[4]
-
$$\boxed{\Phi = \oint \vec{E} \cdot d\vec{A}}$$ [5].
  - You often know the formula for $\oint dA$, so you don't need to evaluate the integral.
-
$$\boxed{\Phi_{\text{net}} = \frac{q_{\text{en}}}{\epsilon_0}}$$ [6].
  - If the electric field lines from a charge *enter and exit* an object, then the flux from that charge is 0.
- A **Gaussian surface** is a closed 3D surface through which flux is calculated.

3.5.2 Deriving the Electric Field a Small Distance Away from a Line of Charge

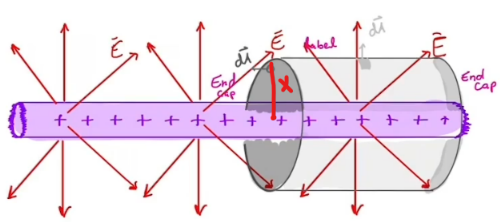

Figure 1: A uniformly charged rod with total charge Q and length L.

[2] Area is a vector because it's defined as magnitude times the normal vector (perpendicular to the surface).
[3] This is not on your equation sheet, but it is a corollary of the equation below.
[4] This is just the definition of a dot product.
[5] Φ isn't written on your equation sheet, but the RHS is.
[6] LHS is not on the equation sheet.

We choose our Gaussian surface to be a cylinder to exploit symmetry. We know $\Phi_{\text{end caps}} = 0$ because it's perpendicular to $\vec{E}$. So, we can just evaluate the flux through the side of the cylinder:

$$\begin{aligned}
\Phi_{\text{side}} &= \frac{q_{\text{en}}}{\epsilon_0} \\
E(2\pi x \cancel{l}) &= \frac{\lambda \cancel{l}}{\epsilon_0} \\
\vec{E} &= \frac{Q}{L(\pi\epsilon_0 x)} \blacksquare
\end{aligned}$$

3.5.3 Deriving the Electric Field a Small Distance Away from a Uniformly Charged Plate

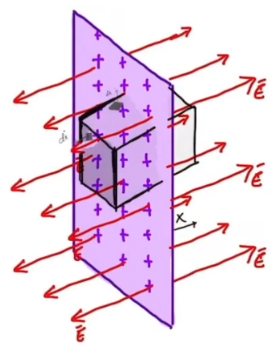

Figure 2: A flat plane with total charge Q distributed uniformly over a plate of area A.

We choose our Gaussian surface to be a rectangular prism to exploit symmetry. Note that only the ends of the rectangular prism contribute to the net flux.

$$\begin{aligned}
\Phi_{\text{end}} &= \frac{q_{\text{en}}}{\epsilon_0} \\
2\Phi_{\text{end}} &= \frac{q_{\text{en}}}{\epsilon_0} \\
2Ea &= \frac{\sigma a}{\epsilon_0}\text{[7]} \\
E &= \frac{Q}{2\epsilon_0 A} \blacksquare
\end{aligned}$$

3.5.4 Deriving the Electric Field a Small Distance Away from a Charged Insulating Sphere

We have to split this problem up into two regions: inside of the sphere and outside of the sphere.

We start with the outside of the sphere. We choose a Gaussian surface to be a sphere to exploit symmetry.

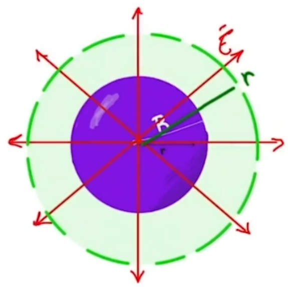

Figure 3: A sphere of radius R and charge Q and Gaussian sphere of radius r.

$$\begin{aligned}\Phi &= \frac{q_{\text{en}}}{\epsilon_0} \\ E(4\pi r^2) &= \frac{Q}{\epsilon_0} \\ E &= \frac{Q}{4\pi\epsilon_0 r^2} = k\frac{Q}{r^2}\end{aligned}$$ [8]

Now, we can work no the inside of the sphere. We choose a Gaussian surface to be a sphere to exploit symmetry.

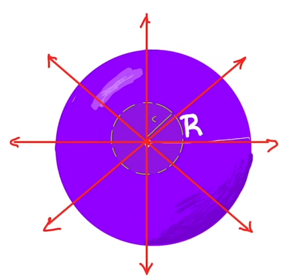

Figure 4: A sphere of radius R and charge Q and Gaussian sphere of radius r.

$$\begin{aligned}\Phi &= \frac{q_{\text{en}}}{\epsilon_0} \\ E(4\pi r^2) &= \frac{q_{\text{en}}}{\epsilon_0}\end{aligned}$$

But q_{en} isn't as easy to find. We know that $\rho = \frac{dq}{dV}$. So, we can arrange and integrate as so:

$$q_{\text{en}} = \int_{V=0}^{V_r} \rho \, dV.$$

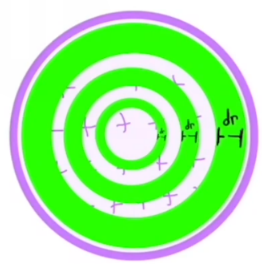

Figure 5: dV is represented by a concentric shell of thickness dr.

Think of dV as a concentric shell with thickness dr. We take each shell and make it bigger and bigger until its radius matches the radius we want. So,

$$dV = 4\pi r^2 \, dr.$$

$$\begin{aligned} q_{\text{en}} &= \int_{V=0}^{V_r} \rho \, dV \\ &= \int_{r=0}^{r} \rho (4\pi r^2) \, dr \\ &= \rho \, 4\pi \int_{r=0}^{r} r^2 \, dr \\ q_{\text{en}} &= \frac{4\pi r^3 \rho}{3} = \frac{Qr}{4\pi\epsilon_0 R^3} \end{aligned}$$

Note that we assumed ρ is constant, but it could be a non-constant function of r.

* * *

We have an inner sphere of radius R uniformly charged with charge $+Q$. An outer conducting shell is placed around it with an inner radius $2R$ and outer radius $3R$ and charge $+q$.

Nothing changes if we're inside the inner sphere or between the sphere and the shell because $\Phi = \frac{q_{\text{en}}}{r}$. Remember that $E = 0$ inside the shell because it's a conductor.

When we're outside of the shell, though, things change.

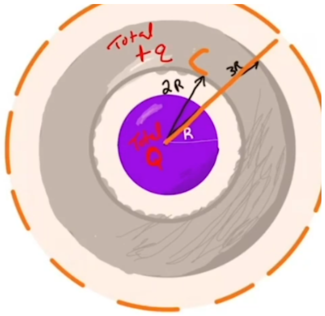

Figure 6: Caption

$$\oint E \cdot dA = \frac{q_{\text{en}}}{\epsilon_0}$$
$$E(4\pi r^2) = \frac{Q+q}{\epsilon_0}$$
$$E = \frac{k(Q+q)}{4\pi\epsilon_0 r^2} \blacksquare$$

3.6 Fields and Potentials for Other Charge Distributions

3.6.1 Dipole

A **dipole** is formed when a positive and negative charge are close together. We'll try to find the net electric field at x.

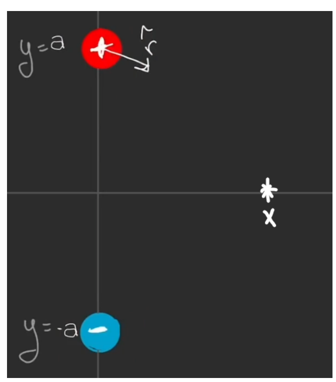

Figure 7: Two particles are located on the y-axis at $y = \pm a$.

We start with the positive charge. We know that $E = \frac{kq}{r^2}$. Then, we plug in

r^2 to get

$$E_1 = \frac{kq}{a^2+x^2}\left(\frac{x}{\sqrt{a^2+x^2}}\hat{x} - \frac{a}{\sqrt{a^2+x^2}}\hat{y}\right) = \frac{kqx}{(a^2+x^2)^{\frac{3}{2}}}\hat{x} - \frac{kqa}{(a^2+x^2)^{\frac{3}{2}}}\hat{y}.$$

Similarly, for the negative charge, we have that

$$E_2 = \frac{k(-q)x}{(a^2+x^2)^{\frac{3}{2}}}\hat{x} + \frac{k(-q)a}{(a^2+x^2)^{\frac{3}{2}}}\hat{y}.$$

Then, we find the net electric field to be

$$E = 0\hat{x} - \frac{2kqa}{(a^2+x^2)^{\frac{3}{2}}}\hat{y}.$$

3.6.2 Charged Rod

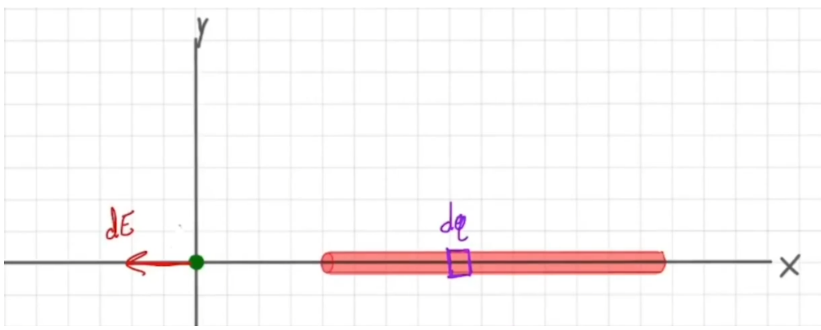

Figure 8: A charged rod of length L lies horizontally a distance A from the origin.

Every infinitesimally small portion of charge dq causes an electric field dE. Then, we have the following:

$$\begin{aligned}
d\vec{E} &= \frac{k\,d\vec{q}\,\hat{r}}{r^2} \\
\vec{E} &= \int_{x=A}^{L+A} \frac{-k\,dq}{r^2}\hat{x} \\
\lambda &= \frac{Q}{L} = \frac{dq}{dx} \\
E &= \int_{x=A}^{L+A} \frac{-k\lambda\,dx}{x^2}\hat{x} \\
E &= k\lambda\left(\frac{1}{L+A} - \frac{1}{A}\right)
\end{aligned}$$

3.6.3 Uniformly Charged Semi-circle

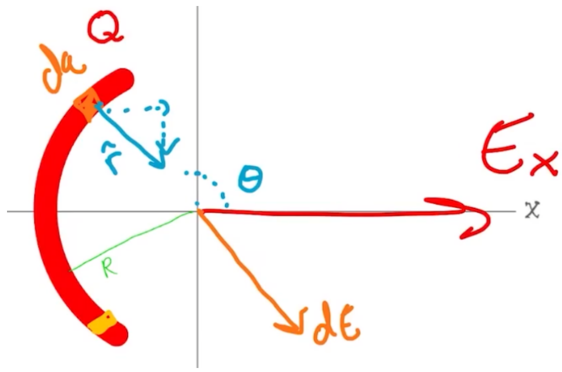

Figure 9: A quarter circle of radius R is uniformly charged with a total charge Q.

By symmetry, the y-components of the electric field cancels out. Then, we find the x-component as follows:

$$\begin{aligned}
dE &= \frac{k\,dq}{r^2}(-\cos\theta\hat{x} - \sin\theta\hat{y}) \\
E_x &= \int_{\theta=\frac{3}{4}\pi}^{\frac{5}{4}\pi} \frac{k\,dq}{R^2}(-\cos\theta) = \frac{k}{R^2}\int_{\theta=\frac{3}{4}\pi}^{\frac{5}{4}\pi} dq(-\cos\theta) \\
\lambda &= \frac{Q}{2\pi R/4} = \frac{dq}{dl}[9] = \frac{dq}{d\theta\, R} \\
E_x &= \frac{k\lambda}{R}\sqrt{2}.
\end{aligned}$$

We can find the potential at the origin, too:

$$\begin{aligned}
V &= \int_{\theta=\frac{3}{4}\pi}^{\frac{5}{4}\pi} \frac{k}{R} dq \\
&= k\lambda \int_{\theta=\frac{3}{4}\pi}^{\frac{5}{4}\pi} d\theta \\
&= \frac{k\lambda\pi}{2}.
\end{aligned}$$

4 Conductors, Capacitors, Dielectrics

4.1 Electrostatics with Conductors

- **Electrostatic equilibrium** occurs when excess charge spreads on the surface of a conductor until there is no more movement of charge.
  - Any excess charge resides on the surface.[10]
  - The electric field inside the conductor is zero.
  - The electric potential inside the conductor is constant and equal to the potential at the surface.
  - The electric field just outside the surface is perpendicular to the surface.
    * For irregularly shaped conductors, this can tell you where the electric field is greatest (places where surfaces curve the most).
- **Conductors** are materials that have many free electrons.
  - According to concept of shielding, in the presence of an external electric field, charges on the surface of a conductor will move until any charges outside the conductor do not affect the electric field inside the conductor.
- $$\boxed{\Delta V = -\int \vec{E} \cdot d\vec{r}}.$$

4.1.1 Touching Two Conducting Spheres

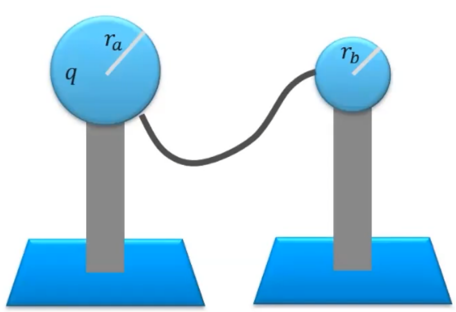

Figure 10: Two conducting spheres are connected with a wire.

We know that $V_a = V_b$ because the wire makes the two conducting spheres functionally a big conductor that's shaped weirdly. Knowing that will solve most of these problems.

[10]Charges only spread out evenly if the conductor is a sphere.

4.2 Capacitors

4.2.1 Introduction

- A **capacitor** is a device that stores and transforms electric potential energy. Capacitance is measured in Farads (F).

- $$\boxed{\Delta V = \frac{Q}{C}}.$$

- $$\boxed{C = \frac{\kappa \epsilon_0 A}{d}}.$$

- $$\boxed{U_C = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2}.$$

- $$\boxed{E_x = -\frac{dV}{dx}}.$$

- $$u_E = \frac{\epsilon_0 E^2}{2}.$$
  - u_E stands for the energy density in an electric field in a vacuum
  - $u_E \propto E^2$.

4.3 Dielectrics

- Dielectrics increase capacitance.

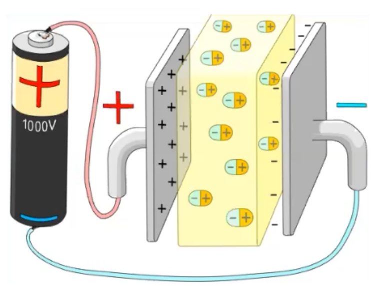

Figure 11: Dielectric placed between two parallel plates. The molecules within the dielectric align with the external electric field.

- When putting a dielectric into a capacitor and a battery is connected, energy increases because the battery supplies it.

- When putting a dielectric into a capacitor and a battery is not connected, energy decreases because the capacitor does work to pull in the dielectric.
- Remember conservation of energy!
- Combine $C = \frac{k\varepsilon_0 A}{d}$, $Q = CV$, $V = Ed$, and $U = \frac{1}{2}QV = \frac{1}{2}CV^2$ together to solve many MCQs.

5 Electric Circuits

5.1 Current and Resistance

5.1.1 Simple Circuits & Ohm's Law

- Conventions say that positive charges move out the positive end of the battery (even though electrons are the ones moving).
- **Current** is the number of charges that pass through in a given unit of time.
  - $I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$.
  - Measured in Amperes (A).
- Voltmeters show potential difference, so they are attached parallel.
- Ammeters show current, so they are attached in series.
- **Ohm's Law**:

$$\boxed{I = \frac{\Delta V}{R}}.$$

  - Only applies for Ohmic devices (follows Ohm's Law).

5.1.2 Current

- In a metal wire, there are lots of free electrons which can move if there's a potential difference across the wire.

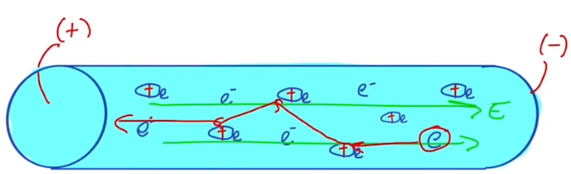

Figure 12: A metal wire of length l experiences a potential difference.

  - The electrons feel an electric force F_e and should accelerate, but don't because it collides with the lattice electrons (atoms fixed in place).

- So $v_{\text{drift}}(v_{\text{avg}})$ is constant and depends on thermal properties of the wire.
- $J = \frac{N}{\text{Unit Volume}} \cdot e \cdot v_{\text{drift}}$, where J is current density, N is the number of charges allowed to flow (free charges), and e is the value of each charge.
- Then,

$$\boxed{I = JA = Nev_d A}.$$

- Electrons flow in the *opposite* direction of current[11].
-

$$\boxed{\vec{E} = \rho \vec{J}},$$

where ρ is resistivity and J is the current density.

  - $J = \frac{I}{A}$.

5.1.3 Resistance & Resistivity

- **Resistivity** refers to the number of collisions an electron has as it travels through a wire. It refers to a material's ability to resist flow.
-

$$\boxed{R = \frac{\rho l}{A}}.$$

- $\rho = \frac{1}{C}$.
- The brightness of a bulb depends on the power it dissipates (and the potential difference).

5.2 Current, Resistance, and Power

-

$$\boxed{P = I\Delta V}.$$

  - You can combine this with Ohm's Law: $\Delta V = IR$.
  - Power's units is Watts.
  - It's useful to combine with $P = \frac{dE}{dt} = \frac{\Delta E}{\Delta t}$.

[11] Because of conventional current and Ben Franklin.

5.3 Steady-State Direct-Current Circuits

- A series circuit gives only one path for current.
  - $$\boxed{R_s = \sum_i R_i}.$$
  - Current remains constant throughout a series circuit. Charge is conserved because there is only one path.
  - Voltage drops across each resistor in a series circuit.
- A parallel circuit gives multiple paths for current.
  - $$\boxed{\frac{1}{R_p} = \sum_i \frac{1}{R_i}}.$$
  - Current will split according to $\Delta V = IR$.
  - Voltage stays the same in a parallel circuit.
- Redraw circuit diagrams with equivalent resistors.

5.3.1 Kirchhoff's Laws

- Use for circuits that have more than one battery or that cannot be reduced to a single loop by combining series and parallel arrangements.

1. The sum of current into and out of a junction is zero: $\sum_{\text{junction}} I = 0$.
   - We define into a junction as positive (+) and out as negative (-).
2. The sum of voltage in a closed loop is zero: $\sum_{\text{closed loop}} V = 0$.

- If loop and current are in same direction, voltage drop. If loop and voltage are in opposite directions, voltage rise. If exiting the positive terminal of a battery, voltage rise. If exiting the negative terminal, voltage drop.

5.3.2 Internal Resistance

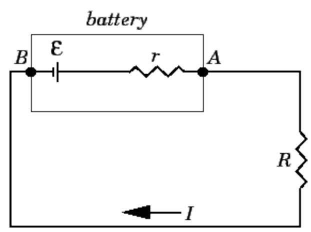

Figure 13: Battery with emf ϵ and internal resistance r.

-
- You can find the internal resistance r of the battery with a Kirchhoff loop that goes from terminal to terminal.

5.4 Capacitors in Circuits

- $$\boxed{\frac{1}{C_s} = \sum_i \frac{1}{C_s}}.$$
  - Capacitors in series have the same charge $Q = C_{\text{eq}} V_{\text{source}}$.
  - Capacitors in series divide the voltage from the voltage source according to $V = \frac{Q}{C_i}$.
- $$\boxed{C_p = \sum_i C_i}.$$
  - Capacitors in parallel have the same voltage.
  - The charge on each parallel capacitor is determined by $Q = CV$.
  - Parallel capacitors have a greater equivalent capacitance and greater total charge, so they store more than capacitors in series.
- When a capacitor is initially uncharged, it acts as a wire. After a long time, it acts like an open switch with charge.

5.4.1 RC Circuits

- The **time constant** τ is the time for a value to rise/fall to $\frac{1}{e}$ of the original value.
  -
  $$\tau = RC^{12}.$$

6 Magnetic Fields

6.1 Forces on Moving Charges in Magnetic Fields

6.1.1 Introduction

- Every magnet has a North and South pole.
- Magnetism is caused by the parallel alignment of electron spins in ferromagnetic materials (materials containing iron).
- Magnetic material can cause domains to become aligned in non-magnetized ferromagnetic materials.
- Outside a magnet, magnetic fields point from North to South. Inside, magnet points South to North.
- The South magnetic pole is the North geographic pole.
- Magnetic fields are represented by $\vec{B}$ with units of **Tesla** ($1T = \frac{N}{a\,m}$) or **Gauss** ($1G = 1 \cdot 10^{-4}T$).

6.1.2 Forces on Moving Charges

- Non-magnetic materials must meet conditions before they feel a magnetic force:
  1. Must have a net charge.
  2. Must be moving.
  3. Direction of the movement cannot be parallel to the direction of the magnetic field.
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  $$\boxed{\vec{F}_M = q\vec{v} \times \vec{B}}.$$
- **Right Hand Rule**:
  - Point your fingers in the direction of the magnetic field (to South).
  - Line your thumb up with the velocity of a positive particle or the current.

[12] Not on the equation sheet, but should memorize.

  - Your palm is the direction of the magnetic force. (If the particle is negative, the force goes the opposite direction.)

- When a particle is moving perpendicular to a magnetic field, we can find its velocity as follows:

$$\begin{aligned}\sum F &= ma \\ qvB &= \frac{mv^2}{r} \\ r &= \frac{mv}{qB}\end{aligned}$$

- Magnetic field never does any work on a particle because displacement and force are perpendicular (recall $W = \int \vec{F} \cdot \vec{dr}$).

6.2 Forces on Current-carrying Wires in Magnetic Fields

- $$\boxed{\vec{F} = \int I\, d\vec{l} \times \vec{B}}.$$

  - $d\vec{l}$ is a segment of wire.
  - Turns into $F = ILB$ in a wire because cross product doesn't matter.

6.2.1 Loop of Current in Magnetic Field

- $$\tau = NIA \times B.$$

  - N is the number of loops of wire, A is the cross-sectional area of the loop, I is the current in the loop, and B is the magnetic field.
  - For torque, wrap your right hand in the direction of the spin and the torque vector will be your thumb.

6.3 Fields of Long, Current-carrying Wires

- $$B = \frac{\mu_0 I}{2\pi r},$$

  where r is the distance from a point to the wire.

- **Right Hand Wire Rule:**
  - Point your thumb in the direction of the current.
  - Wrap your fingers. They will go in the direction of the magnetic field.

- To find the magnetic field at a point surrounded by wires, take the vector sum of each magnetic field contribution.

6.3.1 Parallel Wires

- Currents in the same direction make the wires attract. Current in opposite directions cause the wires to repel.

6.4 Ampere's Law and Biot-Savart Law

6.4.1 Ampere's Law

- $$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I}.$$

6.4.2 Magnetic Field Outside & Inside a Long, Straight, Current-carrying Wire

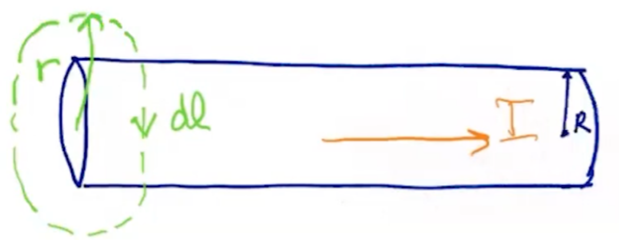

Figure 14: A current-carying wire with an Ammperian loop of radius r and clockwise direction for dl.

We first draw an Amperian loop with radius r and designate a direction for dl. Then, we have the following:

$$\begin{aligned}\oint \vec{B} \cdot d\vec{l} &= \mu_0 I \\ B(2\pi r) &= \mu_0 I \\ B &= \frac{\mu_0 I}{2\pi r}.\end{aligned}$$

Now, we can look at the inside.

Figure 15: A current-carying wire with an Ammperian loop of radius r and clockwise direction for dl.

We first draw an Amperian loop with radius r and designate a direction for dl. Note that the current density is

$$J = \frac{I}{A} = \frac{I}{\pi R^2}.$$

Then, we have the following:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$
$$B(2\pi r) = \mu_0 (\frac{I}{\pi R^2} \pi r^2)$$
$$B = \frac{\mu_0 I r}{2\pi R^2}.$$

6.4.3 Magnetic Field Inside and Outside a Solenoid

Right Hand Rule for solenoids: Wrap your fingers in the direction of the coils and your thumb will be the direction of the magnetic field.

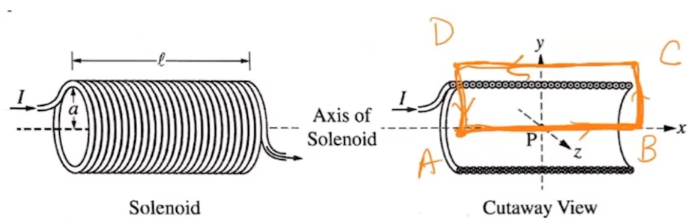

Figure 16: A solenoid with a rectangular Amperian loop.

Note that $I_{\text{enc}} = IN$, where N is the number of loops and $n = \frac{N}{l}$, which is the number of loops per length.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$
$$B_{AB} l + \cancel{B_{BC} l} + \cancel{B_{CD} l} + \cancel{B_{DA} l} = \mu_0 I_{\text{enc}}$$
$$B_{AB} l = \mu_0 I n l$$
$$B_{AB} = \mu_0 n I.$$

BC and DA cancel because $B \perp l$. CD cancels because it's outside of the solenoid, which ideally feels no magnetic force.

6.4.4 Magnetic Field Inside and Outside a Coaxial Cable

We first find the magnetic field when $a < r < b$.

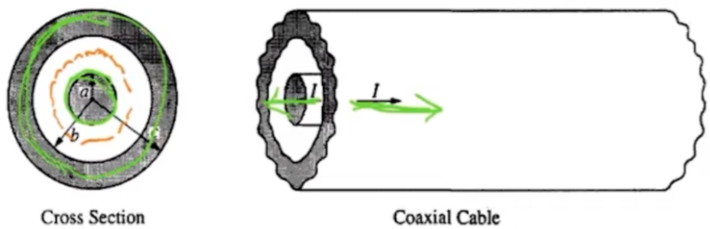

Figure 17: A coaxial cable with Amperian loop and inside and outside current I.

Then, we have the following:

$$\begin{aligned}\oint \vec{B} \cdot d\vec{l} &= \mu_0 I \\ B(2\pi r) &= \mu_0 I B \qquad = \frac{\mu_0 I}{2\pi r}.\end{aligned}$$

We can go inside the inner conductor $0 < r < a$.

$$\begin{aligned}\oint \vec{B} \cdot d\vec{l} &= \mu_0 I \\ B(2\pi r) &= \mu_0 (\frac{I}{\pi a^2} \pi r^2) \\ B &= \frac{\mu_0 I r}{2\pi a^2}.\end{aligned}$$

We can go outside of the cables with $r = 2c$.

$$\begin{aligned}\oint \vec{B} \cdot d\vec{l} &= \mu_0 I \\ B(2\pi r) &= \mu_0 (0) \\ B &= \frac{\mu_0 I r}{2\pi a^2}.\end{aligned}$$

The current goes to the left inside and to the right outside, so it cancels out.

6.4.5 Biot-Savart Law

- $$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}}.$$
- $d\vec{B}$ represents the small individual contributions of circuit elements, r is the distance from a wire, $d\vec{l}$ is a small bit of wire.

6.4.6 Magnetic Field at the Center of a Coil

We can find the magnetic field at the center of a coil of current-carrying wire using the Biot-Savart Law.

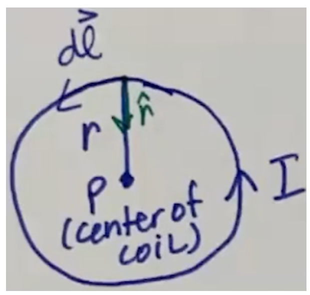

Figure 18: A coil of radius r with counterclockwise dl and I.

$$\begin{aligned}
d\vec{B} &= \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}\\
\int d\vec{B} &= \int \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}}{r^2}\\
B &= \frac{\mu_0 I}{4\pi r^2}\int dl = \frac{\mu_0 I}{4\pi r^2}2\pi r\\
B &= \frac{\mu_0 I}{2r}.
\end{aligned}$$

If there are n loops, then we multiply the whole thing by n: $B = n\frac{\mu_0 I}{2r}$.

6.4.7 Magnetic Field around a Long, Straight, Current-Carrying Wire with Biot-Savart's Law

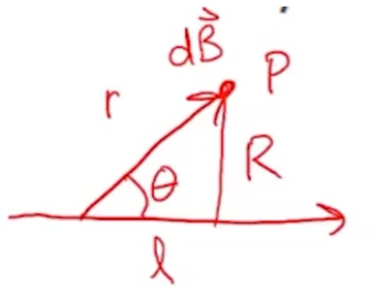

Figure 19: A wire of length l.

$$
\begin{aligned}
d\vec{B} &= \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} \\
\vec{B} = \int_{-\infty}^{\infty} d\vec{B} &= 2\int_0^{\infty} d\vec{B} = \frac{2\mu_0 I}{4\pi}\int_0^{\infty}\frac{dl\ \sin\theta}{r^2} \\
\sin\theta &= \frac{R}{r} \implies r^2 = \frac{R^2}{\sin^2\theta} \\
\tan\theta &= \frac{R}{l} \implies l = \frac{R}{\tan\theta} = R\cot\theta \implies dl = \frac{-R}{\sin^2\theta}d\theta \\
\vec{B} &= \frac{\mu_0 I}{2\pi}\int_0^{\infty}\frac{-R\,d\theta\sin\theta}{\sin^2\theta}\left(\frac{\sin^2\theta}{R^2}\right) \\
\vec{B} &= \frac{\mu_0 I}{2\pi}\int_0^{\infty}\frac{-\sin\theta\,d\theta}{R} = \frac{\mu_0 I}{2\pi R}\int_{\pi/2}^{0}-\sin\theta\,d\theta \\
\vec{B} &= \frac{\mu_0 I}{2\pi R}[\cos 0 - \cos\pi/2] = \frac{\mu_0 I}{2\pi R}.
\end{aligned}
$$

7 Electromagnetism

7.1 Electromagnetic Induction

7.1.1 Magnetic Flux

- Magnetic flux is the amount of magnetic field passing through an area.
- $$\boxed{\Phi_B = \int \vec{B}\cdot d\vec{A}}.$$

7.1.2 Faraday's Law

- Changing the magnetic field through a coil of wire will create a current called the **induced current.**
  - The induced current is caused by an induced emf, ϵ, and $I = \frac{\epsilon}{R}$.
  - The emf is generated by changing the magnetic flux by changing the magnetic field, the area of the loop, or the orientation of the loop in relation to the magnetic field.
-
$$\boxed{\varepsilon = -\frac{d\Phi_B}{dt}}.$$

7.1.3 Motional EMF for a Metal Bar on Metal Rails

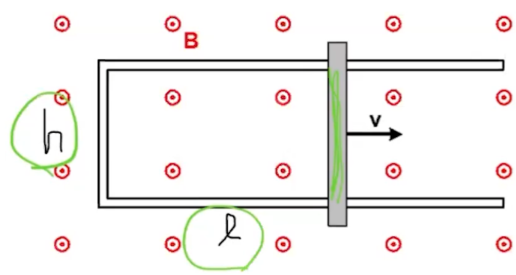

Figure 20: A metal bar moving to the right with velocity v on metal rails with dimensions h and l.

$$\begin{aligned}\varepsilon &= \frac{-d\Phi}{dt}\\ &= \frac{-B\,dA}{dt}\\ &= \frac{-Bh\,dl}{dt}\\ &= -Bhv.\end{aligned}$$

7.1.4 Lenz's Law

- Lenz's Law is used to find the direction of an induced current. It's the reason for the negative in Faraday's Law: $\varepsilon = -\frac{d\Phi_B}{dt}$.
  - The induced current creates its own magnetic field that opposes the change in the magnetic flux.
- How to apply Lenz's Law:

1. Determine the direction of the magnetic field in the conducting loop.
2. If the flux is INCREASING, the induced current must create a magnetic field in the OPPOSITE direction of the current magnetic field.
3. If the flux is DECREASING, the induced current must create a magnetic field in the SAME direction of the current magnetic field.

7.1.5 Forces and Torques

- Wires and loops can experience net forces and/or net torques because of changing flux.

7.2 Inductance

- **Inductors** are coils of wire that can self-induce an emf and current. A coil is said to have inductance

$$L = N\frac{\Phi_B}{I}.$$

  - Unit of inductance is the Henry (H).
  - Inductors oppose any change in the current of a circuit. Larger inductors can slow a change in current more.

- $$\boxed{\varepsilon = -L\frac{dI}{dt}}.$$

- Faraday's Law to inductors:

$$\varepsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}.$$

- $$\tau = \frac{L}{R}\text{[13]}.$$

- At first, an inductor doesn't let any current through. After a long time, an inductor acts like a wire.

7.3 Maxwell's Equations

1. Gauss's Law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}.$$

2. Gauss's Law for Magnetism:

$$\oint \vec{B} \cdot d\vec{A} = 0.$$

[13] Not on the equation sheet, but should memorize.

3. Faraday's Law:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}.$$

4. Ampere's Law[14]:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I.$$

[14] But this is inaccurate, since an electric field induces a magnetic field which induces an electric field... We have to add $\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$.